



CONVOLUTIONAL

CODES

A class of
channel codes or
Error control codes.

Consider a bit sequence,

$x(0), x(1), x(2), \dots$
possibly infinite

Example of convolutional code

$$\begin{aligned} C_0(j) &= x(j) + x(j-2) \\ C_1(j) &= x(j) \oplus x(j-1) + x(j-2) \end{aligned}$$

code bit 0 at time j

code bit 1 at time j

Binary Field Arithmetic

At Each time instant
2 code bits are generated

$$\Rightarrow \text{Rate} = \frac{1}{2}$$

Binary Field Arithmetic

Similar to XOR operation {

$$\begin{array}{lcl} 0 + 0 & = & 0 \\ 0 + 1 & = & 1 \\ 1 + 0 & = & 1 \\ 1 + 1 & = & 0 \end{array}$$

AND operation

$$0 \cdot 0 = 1 \cdot 0 = 0 \cdot 1 = 0$$
$$1 \cdot 1 = 1$$

+ , ·

operations . commutative

Properties:

$$\begin{array}{l} a + b = b + a \\ a \cdot b = b \cdot a \end{array}$$

Associative:

$$a + (b + c) = (a + b) + c$$

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

Distributive:

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

$$(a + b) \cdot c = a \cdot c + b \cdot c$$

Is this code Linear?

Consider 2 information bit sequences.

$x(0), x(1), x(2), \dots$

$y(0), y(1), y(2), \dots$

$c_0^x(j) = x(j) + x(j-2)$

← coded bit 0 for sequence x

$c_0^y(j) = y(j) + y(j-2)$

← coded bit 0 for sequence y

$$C_0^x(j) + C_0^y(j)$$

$$= x(j) + x(j-2) + y(j) + y(j-2)$$

$$= (x(j) + y(j)) + (x(j-2) + y(j-2))$$

From commutativity
& associativity

$$= C_0^{x+y}(j)$$

← coded bit for added information sequence

$$C_0^x(j) + C_0^y(j) = C_0^{x+y}(j)$$

code:
Bit 0

code followed
by addition

Addition of
information bit
sequences followed
by code.

code:
Bit 1

Both Equal.

$\Rightarrow$ code is Linear

$$C_1^x(j) + C_1^y(j) = C_1^{x+y}(j)$$